

CHAPTER TWO

THE BEGINNINGS:

THE FIRST TWO CONFERENCES

The *International Mathematical Union* (IMU) created the *International Commission on Mathematical Education* (ICMI). This Commission sponsored the *Interamerican Conferences on Mathematics Education* summoned for the purpose of discussing the problems posed by Mathematics teaching in the various countries of the Americas.

The first two conferences were of extraordinary importance and greatly influenced the teaching of Mathematics in participating countries. This influence was a consequence of the clear definition of the main goal of both events: to introduce into the curricula (especially in secondary schools) the subjects, language and methods of "Modern Mathematics".

THE FIRST CONFERENCE

The *First Interamerican Conference on Mathematics Education* was held in Bogotá, Colombia, December 4-9, 1961. It was sponsored by the *International Commission on Mathematical Instruction* and the Organization of American States (OAS), and was attended by mathematicians and Mathematics teachers, as representatives or guests, from 23 American countries. Some of these professionals, as well as distinguished European mathematicians, were invited to lecture on Modern Math and also on how to teach it and promote its acceptance.

The main purpose of this *Conference* was to explore the methods of teaching Mathematics at the secondary school level and also in colleges and universities, and to pass resolutions with a view to future cooperation. More specifically, the intention was to introduce in Latin America the reform of Mathematics teaching (at the secondary level) that was going on in many countries, especially in Europe and the United States. As explained in the preceding chapter, this reform was a worldwide movement that began in the 1950's, whose purpose was to reform Mathematics syllabi and curricula that were in force at the secondary level. The reform was initiated in the developed countries, especially the United States and France, and was born as a response to a problem that, at that time, was considered to be critical: the need to bridge the gap between Mathematics as practiced by researchers and professionals in the field, on the one hand, and, on the other, the type of Mathematics then taught in secondary schools. The concepts that were to be introduced in elementary and secondary schools were not precisely the connections between Mathematics and the natural sciences, nor Discrete Math, but set theory, abstract algebraic structures, and unifying and universal concepts. The purpose was to give unity to

Mathematics, using sets, relations, functions and operations as basic concepts, as well as fundamental structures of groups, rings, fields, and vector spaces. The need to adopt modern symbolism was also established. Thus, the main objective of this *Conference* was to foster these approaches among the delegates and to reach a commitment from them, asking that they promote curricular changes in their respective countries.

The opening address was given by Dr. Marshall Stone, President of the *International Commission on Mathematical Instruction*, who gave a brief summary of the process of implementing Modern Math in European and U.S. secondary schools.

THE IDEAS

The main ideas brought forward in this *Conference* were the following:

- (a) The need to change the way of teaching Geometry in Secondary Schools: to teach Geometry from the perspective of Linear Algebra, forsaking Euclidean Geometry.
- (b) The need to teach Mathematics, in general, through the study of the fundamental structures, with the purpose of underscoring their unity. In this area, the teaching of Modern Algebra became of paramount importance.
- (c) The above goals could only be achieved if, at the same time, a well-organized plan was carried out that was oriented to the in-service training of teachers, thus preparing new mathematics teaching professionals with the ideas of the reform, as well as improving research in mathematics
- (d) The above goals could not be achieved without a parallel plan, very well organized and aimed at the training of professors who were currently teaching. The aim was to instill these ideas in new teachers of Math, and also to upgrade Mathematics research.

As far as the first of the above points is concerned, Professor Howard Fehr, from the United States (who had represented his country at the 1958 Edinburgh Conference), set forth the major ideas with his contribution: "Reforming the Teaching of Geometry". He gave a brief account of the development of Geometry, emphasizing that, in spite of the new developments that took place at the turn of the century, which indicated new directions, Euclidean Geometry still being taught in secondary schools. Fehr was very much against this state of affairs, up to the point of stating that "Euclidean Geometry (...) has nothing to do with these subjects; nowadays it is sterile, outside the main course of mathematical advancement, and it can be filed in the archives, without any fear, for the benefit of future historians".¹

Fehr, in his address to the Conference, critically questioned the teaching of Euclidean Geometry in secondary schools, and strongly supported the thesis advanced by Dieudonné in Royaumont. Moreover, Fehr proposed a program for teaching geometry in secondary schools. He said that it was possible to teach the essentials of Euclidean Geometry in two or three months. Thereafter, additional deductive work in Algebra should be given to the

students, including new number systems and algebraic structures; finally, a combination of Algebra and Geometry through the study of affine plane Geometry. All of this was in line with the ideas proposed in Royaumont by Dieudonné and Choquet, and with the viewpoints expressed by Henri Cartan at the Bologna meeting. In other words, the purpose was to guide the students, as rapidly as possible, towards the study of vector spaces. However, in spite of all this, Fehr himself cautioned that, at this level, axiomatic knowledge should not be given too much emphasis.

Challenging this position, in the debate following Fehr's presentation, the delegates from several countries voiced their doubts. For instance, Professor Catunda (Brazil) disagreed with Fehr's vision and asked if in his country it would not be convenient "at least Euclidean Geometry", and Coleman (Canada) explained that, in his own country, the reason to teach Euclidean Geometry was that "whoever has developed an interest in Mathematics found in Euclid his or her first incentive". In general terms, however, the discussion evidenced a certain degree of agreement (arising mainly from the European guests) with the ideas advanced by Fehr. Choquet agreed with Fehr, although favoring the introduction of axioms, and Pauli (Switzerland) said that the ideas advocated by Fehr had been implemented in his country for the last ten years.

Fehr's presentation along with the debate that we have summarized is very illustrative of the objectives of this *First Conference* and the doubts that were still present in that moment. As can be observed, the ideas of both Choquet and Pauli contributed to develop the criterion for change in the teaching of Geometry, despite the resistance of some of the participants such as Catunda and Coleman who expressed their doubts with at least this aspect of the reform.

The second of the ideas, indicated in (b) above, was present throughout the conference, especially in the presentations of those invited from France and the United States. This can be seen as much in their presentations as in their interventions in the debates. In this regard two of the presentations were very significant. That of Choquet (France) entitled "The New Math and its Teaching" and that of Marshall Stone called "Some Characteristic Tendencies in Modern Mathematics".

In the first of those presentations, Choquet began by giving a quick overview of modern Mathematics and then his view of what mathematics should be included in secondary education. His opinion was that teaching at all levels should be revised to reflect the discovery of the fundamental structures, given that as we move toward an increasingly greater unity of mathematics we should also move to a unity in its teaching at all levels. He said: "Our lemma will be: algebra and the fundamental structures from the School to the University."²

An interesting detail: Choquet added that all teaching based on the historical method had become inconceivable. His entire discourse pointed out the need to put the student in contact as soon as possible with the unifying concepts and fundamental structures. He noted the need for the mathematician, giving little importance to psychopedagogical considerations. For example, he expounded the following principles:

- “1. We should accustom our students to think in terms of set and operations as early as possible. It will be necessary to teach the simple, universal, and precise language of sets. At the same time we should teach them the rudiments of logic in its relation to the grammatical study of their language (to negate a proposition, to understand the force of the words and, or, for all, there exists).*
- 2. At a very early age, our students should have a clear understanding of the concept of function. They should be able to construct various examples of functions in arithmetic, algebra, physics, and to produce the composition of two functions, to take the inverse function of a bijective function, to recognize a group of transformations.*
- 3. The students should be able to recognize at an early age the relation of equivalence (numerous examples; quotient-sets), relations of order, and they should study some concepts of topology.*
- 4. In all fields, it will be necessary to get directly at the essential tools that have numerous and immediate applications”³*

Marshall Stone complemented these ideas by proposing, as something of great importance, the development of the basic elements of Modern Algebra in secondary teaching. He indicated that it seemed possible to teach Modern Algebra at the secondary level up to the point of treating polynomial rings in a field. Nevertheless, in the debate that followed, doubts were expressed. Professor Laguardia pointed out a fundamental aspect that had not been taken into consideration: How to take into account the psychological development of youth? There was no adequate response to his question. Thus, there remained a doubt with respect to the relevance of the reform, at least in the form in which it was being suggested.

It can be said that those two presentations are a reliable representation of the ideas that the organizers of the *Conference* had in mind. However, many of the other presentations were along the same lines, although, perhaps, not with the same clarity in their thinking.

Among them were the presentations of the Latin American educators. The first of those was from Professor Alberto González Domínguez of Argentina entitled "Mathematics and our Technological Society". Professor González expressed some ideas about the relations mathematics-physics, mathematics-automatization, and the importance of mathematics reasoning for approaching many technological and scientific problems. His interest was in making that point, but he did not propose any initiatives for the teaching of mathematics.

Another presentation along the same lines was given by Professor Enrique Cansado of Chile and was called "Modern Applications of Mathematics". He mentioned some of the applications of mathematics such as operations research, linear programming, the simplex method, nonlinear programming, dynamic programming, game theory, etc. His thesis was that these theories, at least in their elemental level, should be introduced into secondary teaching. However, in the debate that followed, some of the participants, especially the Europeans Choquet (France) and Bongaard (Denmark), suggested that there were more

interesting and important topics for secondary mathematics, namely, those topics mentioned above.

It should be said that not everyone was in agreement with the main ideas being presented; at least as they were conceived. In general, some presentations implied the need for change, but for many of the participants it appeared that the changes being proposed were too radical. For example, in his talk on "Some Ideas about the Teaching of Mathematics in the University", Professor Guillermo Torres (Mexico) expressed his doubts about what should be taught and how it should be taught. His thesis was that you could not just abandon entire topics from classical mathematics, as it would then be possible to fall into formal definitions and concepts that would communicate absolutely nothing to students, given that they would not be familiar with specific cases that are more concrete. He indicated that the new ideas that students acquire should be accepted by them as something natural. He further suggested that mathematics should be taught by more or less following its historical development. This was a focus in opposition to that expressed by Choquet (who said that teaching based on the historical method was inconceivable). Countering that, Torres claimed that the presentation of mathematics in its exclusively formal aspect "makes it appear to be an inhuman activity and with no sense at all," even though that was the style that was being imposed more and more.

The last of the main ideas of the *Conference* was of a more operational nature. It is obvious that no reform can be carried out without adequate preparation of personnel that are in direct contact with the students and are putting into practice the teaching of so many new concepts (and old ones too but with a new language and organized differently). Therefore the professional development of teachers who would carry out the reform was very important.

Thus, two of the presentations were on the preparation of mathematic teachers and were given by Latin American professors: A. Valeiras and Luis Santaló (Argentina)⁴, "The Formation of Mathematics Teachers", and Omar Catunda of (Brazil), "The Preparation of Mathematics Teachers". These presentations and the debate that followed were very important because they made clear a situation with respect to teaching mathematics in Latin American countries (which is very similar to what happens still): a lack of fully-trained teachers, inadequate preparation, difficulties of support and professional development, etc. In this respect the statement of Prof. Catunda is very illustrative: "the formula that I would shout for Brazil is not 'Down with Euclid', but 'At least Euclid'⁵".

There were also presentations that offered information on and analyzed mathematics programs in countries where reform efforts were already underway. They served to support the ideas presented in the talks in favor of the reform.

Among them were the following: "New Tendencies in the Teaching of Mathematics in Colleges in the United States", Professor E.J. McShane (USA); "The Mathematics Programs in Swiss Secondary Schools, Professor Laurent Pauli (Switzerland); and "The Mathematics Program in Denmark", Professor Sven Bungaard (Denmark). They presented experiences on the teaching of mathematics in their countries.

The address of Professor E.G. Begle (USA), "The Reform of Mathematics Education in the United States", indicated how reform of mathematics teaching was being carried out in that country. He explained the predominant role of the *School Mathematics Study Group* (SMSG) in its efforts for improving the school program by providing materials and guidelines for the preparation and in-service training of teachers, as well as the strong financial support provided by the National Science Foundation (NSF).

The last address was given by Professor Schwartz (France) on "The Role of Mathematics in Physics from the Point of View of Scientific Education".

RECOMMENDATIONS

The crystallization of the main ideas expressed and discussed was presented in the *resolutions* of the *Conference*. They were divided into three areas:

- I. Preparation of Teachers.
- II. In-Service Teachers.
- III. Improving teaching.

To better understand the scope of that *First Conference* it is necessary to consider some of those resolutions.

“ I. *In connection with training of teachers,*

1. *That centers for the training of high school mathematics teachers should offer scholarships and other facilities to those students who choose this career and that high school students should be informed, by means of conferences and publications, of the existence of a career as teachers and researchers in this field, and of the social importance and of the possibilities offered to those who follow it.*
2. *That the training of teachers of mathematics should be the sole responsibility of the university and under the influence of the most competent mathematicians, to avoid the cleavage between the teaching of mathematics and progress in science and technology. In the meantime, where this training is carried out in special institutions, mathematics courses should be of a university level.*
3. *That in the training of teachers of mathematics in the secondary schools, the courses should be modernized and those of a pedagogical character should be limited to proper proportions.*

II. *In connection with teachers in active service,*

4. *That regular contact be maintained between high school teachers and university professors, encouraging the former periodically to attend courses for improvement (regular or special), and that the means to achieve this end, such as scholarships at home or abroad, be increased.*

5. *That steps be taken to raise the socioeconomic level of the secondary school teacher holding a regular certificate, such as:*
 - (a) *Guarantee tenure.*
 - (b) *Establish basic salaries equal to those of other professions requiring similar or equivalent academic preparation.*
 - (c) *Establish a system of promotions with its corresponding implications (increase in salary, reduction of working hours, etc.) automatically based on the number of years of service, considering supplementary advantages and taking into account publications and activities aimed at self-improvement.*
 - (d) *Establish the sabbatical year.*
 - (e) *Offer the teacher the possibility of a regimen of complete dedication, as a favorable condition necessary to his progress.*
6. *That a maximum of incentives be assigned (scholarships, compensation, etc.) so that the teachers of the secondary school who are without certificate but are in active service can obtain one, and therefore can be covered by the system established in article 5 either by completing their university studies or by taking special courses created for this purpose.*

III. In connection with the improvement of teaching,

7. *That the realization of courses and the creation of institutes of an experimental character, for trying out new texts and new methods of teaching mathematics, be encouraged.*
8. *To suggest to the International Union of Mathematicians, UNESCO, and the Organization of American States, to take under consideration the following steps:*
 - (a) *The intensification of programs for the training of secondary school teachers of mathematics.*
 - (b) *The dispersion of activities, projects, and publications which have to do with the improvement and modernization of the teaching of mathematics.*
 - (c) *The publication and distribution of reports, new texts, and translations written for teachers of the secondary schools for their use in teaching and in self-improvement.*
 - (d) *The encouragement of research as an avenue for scientific and technological progress and as a factor in motivating teaching.*
 - (e) *The creation of an international center for the purpose of collecting and disseminating information that is relative to new experiments and new ideas in mathematics education.*
9. *To promote a wide exchange of information on new ideas in the teaching of mathematics in all countries through national meetings and other international conferences such as the present one."*

THE FIRST COMMITTEE

The most important resolutions for our purposes in this book were resolutions that proposed:

“The creation of an Inter-American Commission on Mathematics Education, of a permanent character, for the purpose of providing continuity to the projects and ideas discussed in this Conference and to promote action calculated to raise the level and efficiency of secondary school and university teaching of mathematics.”

It was also recommended that:

“That delegates and participants establish and maintain contact with the authorities of their respective countries, so that effective measures can be taken to put into practice these recommendations.”

The *Conference*, in one of its resolutions, designated the following individuals to act as a *pro tempore* committee until the *Interamerican Mathematics Education Commission* was established, according to recommendations in the document:

Marshall Stone (USA) *President*,
Alberto González (Argentina),
Bernardo Alfaro (Costa Rica),
Alfredo Pereira (Brazil), and
José Tola (Peru).

As can be observed, the recommendations of this *Conference* were of great importance because they committed the delegates from each country to the process of reform. They would be promoted on two fronts: on the one hand, the delegates would try to get their governments to reform the mathematics programs at the secondary level to carry the stamp of modern mathematics. On the other hand, they committed themselves to trying to influence universities and teacher training institutes to do in-service training and to prepare new mathematics educators with the ideas of the reform. The resolutions adopted indicated the success obtained by the organizers of the *Conference*, at least in the aspect of starting a machine for reform in Latin American countries. Apparently the reluctance shown in some cases was smoothed over.

From another perspective, as can be seen in the list presented below of sponsors of the event, that *Conference* had the support of international and other organizations, especially from the United States, which were interested in having the ideas of reform in mathematics teaching realized in all the countries of the continent. This is evidence of the great concern in this matter from the highest levels, and the pressure that possibly was brought to bear so that the recommendations were approved in the way they were presented.

THE SECOND CONFERENCE

The *Second Interamerican Conference on Mathematics Education* was held in Lima, Peru, December 5-12, 1966, that is, five years after the *First Conference* was held in Bogotá. If the *First Conference* served to promote the introduction of the teaching of modern mathematics in American countries, the *Second* one had as its main focus the analysis of the progress of reform. That focus was declared in the opening address by Marshall Stone.

Besides the invited speakers who presented general topics related to mathematics teaching, the Organizing Committee of that *Conference* asked the delegates from each of the participating countries to present a report in which they summarized the efforts realized in their countries during the period between the two *Conferences* towards the objectives outlined in the *First Conference*.

In his opening address, Dr. Stone recognized the scope of the problem of teaching mathematics, as well as the difficulty of solving it in a practical manner. However, at the same time, he indicated that the Organizing Committee selected a restricted number of topics so that they could be discussed throughout all the activities.

THE MAIN TOPICS

The theme to be studied in the *Conference* was posed in the following form:

*“In the first place, it is natural that we wish to review what has taken place in the hemisphere since the first Inter-American Conference on Mathematical Education, held almost exactly five years ago, in Bogota, Colombia. We must now ask: ‘What had the report of that conference to do with what has taken place in the last five years? Have its recommendations had any influence at all? Have some of them proved to be less practical than we had supposed at the time when we formulated them? In which countries has progress been most marked? In which countries have especially difficult problems been conquered?’ So we should now look back on these five years and, through the medium of a number of reports, and the discussion of them, try to see what the impact of the first conference has been and what we have succeeded in accomplishing all over the hemisphere during that time.”*⁶

Two topics of great importance were also proposed: in the first place, the problem that students face in moving from secondary schools to the universities, and, secondly, the preparation of teachers for the primary and secondary levels. In the first of these topics, the difficulties that students face when going to a higher level of education were recognized. Generally deficient preparation causes many difficulties in adapting to the new styles of teaching that are present in higher education. The second topic was recognized to be of

great importance in order to have success in any attempt to reform the teaching of mathematics.

Thus, the tasks of the *Second Conference* were dedicated to the following three topics:

1. To review what has taken place in the hemisphere since the first Inter-American Conference on Mathematics Education.
2. The problem posed by the students' passage from the secondary school into the university.
3. Preparation of teachers for service in the primary and secondary schools.

The presentations were on those topics, and, also, on the problems that were arising in the implementation of the reform of mathematics teaching in the various countries of Latin America. The presentations were divided into four blocks:

- A. On Problems in Mathematical Education in Latin America.
- B. On Mathematics Improvement.
- C. On Curriculum and Transition.
- D. On Teacher Education.

Within the topic dealt with in Block A, various problems were indicated. Some of them were related to the sociocultural and economic characteristics of the Latin American countries. Others were more specific to bringing about a reform.

Professor Rafael Laguardia of Uruguay made several observations about the first type of problems in his address. His conclusions indicated the existence of several obstacles that impeded the development of mathematics and other basic sciences in Latin America. In particular, he highlighted two elements: the illiteracy that existed in almost all Latin American countries, and the rapid population growth that obliged the use of teachers without the necessary preparation in the teaching of mathematics. Thus, he proposed that the reform should be initiated, at least in his country, only in the higher levels of secondary education and that the universities should actively participate in the reform process. He added that not only was the participation of educational researchers necessary, but also that of mathematics researchers. To all of that Laguardia added the need for centers of scientific research and higher education to collaborate closely with the reform in mathematics teaching.

With respect to the progress of the reform in mathematics teaching and possible solutions, Professor Luis Santaló referred to some of the problems that the reform had encountered in Latin America, specifically with teachers and programs. He identified a series of problems that had occurred during the reform process, some of which he said had been foreseeable and others, perhaps unexpected, that had arisen during the process.

Given the historical importance of Professor Santaló in mathematics in the Americas, it is interesting to mention with some detail the problems he referred to:

- Difficulties in convincing teachers of the need and possibility of reform. To overcome that difficulty he proposed some measures such as: convincing teachers about the recommendations of the meeting and congresses that had been held on the matter, the extensive use of modern mathematics in university level texts, the temporary displacement of the classic programs of mathematics, a great part of the mathematics being taught should be taken out of official programs. On the other hand, he proposed that teachers do a survey that would let them know that many of the topics that they had been teaching were never used, thus there was no practical argument to retain those topics. To determine the formative value of such topics, they should be analyzed in terms of reasoning as opposed to routine, and then compared with modern topics.

- A second problem was convincing parents, i.e. the problem of convincing public opinion. Here Santaló proposed that applications of mathematics not be lost from view, but used to motivate some of the modern topics in connection with science.

- Another problem was the preparation of teachers and student textbooks. He proposed the introduction of modern mathematics into teacher training institutes so that their graduates would be able to teach reform-based programs. He also proposed in-service teacher training. With respect to textbooks he considered that the only way to resolve the situation was to publish student textbooks with a modern mathematics focus.

- The last of the problems identified by Santaló referred to the difficulty of changing Ministry of Public Education regulations in the Latin American countries. That increased the difficulty of having experiences with the new programs. He pointed out, as his last point, an aspect to which he did not give much importance: teachers who did not understand very well what was intended by the reform and, with enthusiasm, taught their courses "full of trivialities, and conceptual errors, sowing general confusion".

The address of the Peruvian professor, José Tola, considered the problem of the development of mathematical research in Latin America. He felt that much had been done in the area since 1961 and therefore mathematicians were busy with research and were not available to work on carrying out the reform. He concluded by calling for the creation of the conditions necessary for the preparation of mathematicians and mathematics researchers who could serve as support for the reform in mathematics teaching. In that respect he recommended: increasing the number of candidates and improving selection procedures; strengthening schools of mathematics in the universities, sending many students abroad for study; and insuring adequate conditions when graduates of foreign universities returned to their countries of origin.

In relation to the topic of the progress in Mathematics Teaching, four addresses were given. Three of them reported on the progress made in the reform of mathematics teaching in a few countries (Spain, Chile and Brazil), and the fourth discussed activities of the OAS with respect to Mathematics.

Professor Pedro Abellanas presented some of the progress in the reform of mathematics teaching in Spain. He mentioned holding annual meetings, since 1960, with

secondary school teachers and university professors, in which there were discussions on teaching modern mathematics. As a result, various studies were carried out and thus the program in the Licenciatura in Mathematics had been modified. He reflected on the importance of the teaching of secondary mathematics, especially in its formative aspects. He indicated that several courses had been organized for mathematics teachers which treated topics such as proportions, similarity, measurement, natural numbers, whole numbers, rational numbers, polynomials, irrational expressions, etc. Later he proposed a mathematics program for each year of secondary education.

In his address, Professor César Abuaud presented some of the progress in mathematics in Chile. He mentioned some positive aspects such as: the wide diffusion of the recommendations from the Conference in Bogotá, the work of the SMSG group, and the close contacts with the "spirit of renewal in Europe". He also provided a list of the steps that had been taken since 1962.⁷

Professor Osvaldo Sangiorgi gave information on some of the progress in mathematics teaching in Brazil: more unity among the universities, institutes and other groups; increased cooperation among university mathematicians and secondary school teachers; creation of new mathematics departments in various universities; increase in the number of teacher training centers; extraordinary increase in the number of teachers taking in-service courses in mathematics; increase in the number of secondary teachers with university degrees; realization of congresses, colloquia, and other activities dedicated to the teaching of mathematics. In conclusion he presented the new program that was being developed for secondary schools in Brazil.

Andrés Valeiras, representative from the OAS, presented the contributions of the OAS in improving the teaching of mathematics. The Department of Scientific Affairs increased its technical assistance activities to achieve the following objectives:

- a. Aid to Ministries of Education in their tasks of modernizing the curriculum.
- b. Aid to Ministries of Education and schools in offering training for in-service teachers and in modernizing the teacher training curriculum.
- c. Aid to encourage research.⁸

That report made it very clear that the OAS had participated significantly in the reform of mathematics in Latin America. Among the activities were the establishment of summer institutes, interchange of scientists, meetings, scholarships, studies on the teaching of science and engineering, and various publications. It is worthwhile to give some details on what was developed:

Scholarship Program. Scholarships for citizens of member nations for high level studies in other American nations.

Programs of Direct Technical Assistance. Visits from expert consultants.

Exchange Programs. Financing of visiting professors in universities in the member nations.

Program of Integrated Projects. Initiatives planned to give training, technical assistance, equipment, etc., to institutions, universities, etc., in Latin America.

Technical Cooperation Program. Training of personnel.

Special Programs:

Annual summer institutes in the USA.

Summer institutes in Latin America.

Regional meetings.

The Study on the Teaching of Science and Engineering in Latin America.

A Guide to Scientists and Scientific Institutions.

Publications.

In Block C, new curricula, as well as an analysis of the state of reform in each country, were presented.

In this context the following individuals related their experiences: Howard Fehr (USA), Carlos Imaz (Mexico), Erik Kristensen (Denmark), Eugene Northrup of the Ford Foundation (Turkey), Georges Papy (Belgium), Andrés Revus (France) and Eduardo Suger Cofiño (Guatemala).

In each case there was talk about the way in which proposals for the implementation of modern mathematics at both the pre-university and university levels were being carried out. The diversity of the countries represented in the presentations denoted the interest in propagandizing the worldwide nature of such reform and the need to know how it was being carried out in other latitudes.

In Block D, dedicated to the preparation of teachers (achievements and difficulties in various countries), there was participation from Mariano García (Puerto Rico), Martha María de Souza Dantas (Brazil), Hans-Georg Steiner (Germany), and Luis Santaló and Renato Völker (Argentina). Here the particular experiences of each country were presented.

NATIONAL REPORTS

A second part of the *Conference*, which was very important to its objectives, was dedicated to reports from the various participating countries concerning the progress of reform. A total of 22 delegations presented their reports: Argentina, Bolivia, Brazil, Canada, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Haiti, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Puerto Rico, Trinidad and Tobago, United States, Uruguay, and Venezuela.

It is interesting to point out that a caution was given that the reports were "informal" in the sense that they were not given by officials from the governments, but were simply the appraisals of the participants of each country.

Some countries showed more progress in the process than others. For example, from the reports of Argentina, Brazil, Canada, and the USA it is obvious that the process began with various aspects of reform, including profound changes in the contents of secondary and university mathematics programs, using new approaches to the pre-service and in-service teacher education. In other countries, such as Chile and Costa Rica, new mathematics programs were promoted in all secondary schools. In some other countries such programs were beginning to be used in just a few institutions as a pilot project, such as was the case in Ecuador. Thus, the majority of the countries reported making at least some attempts to introduce changes. Nevertheless, there were some countries, such as Bolivia, in which it had not been possible to implement any changes. In general, partial or total changes were carried out at the secondary level; in some cases there were changes in teacher preparation programs; and in many there had been training sessions for teachers.

Something important: there was an effort in most countries to produce their own *textbooks*, following the guidelines for teaching modern mathematics in accordance with the directives of the *First Conference*.

It should be mentioned, however, that the majority of the delegates talked about the problems that they had encountered. In general, those problems were common in most countries, such as the difficulty in pre-service and in-service teacher training, and the slight possibilities to carry out the reform with success (at least in the short term) in every country, because of a shortage of both human and economic resources. In many of the countries teacher preparation was deficient. The expansion of the educational system was such that, in many of the countries, many of the teachers who taught mathematics had an inadequate preparation, or, in many cases, none at all. It can be said that this was perhaps the main problem encountered in the reform process.

THE CONCLUSIONS OF THE CONFERENCE

One of the important parts of the *Conference* was a section of *conclusions* in which goals were proposed for continuing the reform of mathematics teaching in the American countries.

During the period from 1961 to 1966 the *pro tempore* committee that had been elected in the *Bogotá Conference* functioned as the executive committee of the *Interamerican Committee on Mathematics Education* (IACME). That committee had some difficulties, mainly economic, in functioning efficiently. Those difficulties permitted the committee to meet only occasionally. Therefore in the *Lima Conference* some basic norms were proposed that would permit the Committee to function more efficiently. The norms that were approved in the closing session on the 12th of December of 1966 were:

“A. The Inter-American Committee on Mathematical Education (IACME) originating in the First Inter-American Conference on Mathematical Education, December 4-9, 1961, is a non-governmental body affiliated with the International Union of Mathematicians through the International Commission on Mathematical Instruction. The purposes of the Committee are to serve as a technical organ in the sense of, and the scope of, the recommendations made at the Conference cited above and at the Second Inter-American Conference on Mathematical Education held at Lima, December 4-12, 1966.

B. In accordance with the decisions of the Lima Conference, the composition of the Committee, until the next conference is held, shall be:

*Marshall H. Stone, U.S.A., President
Cesar Abuauad, Chile
Ricardo Losada, Colombia
Manuel Meda, Mexico
Leopoldo Nachbin, Brazil
Luis A. Santalo, Argentina
Juan Jorge Schaffer, Uruguay
Egardo Sevilla, Honduras
Jose Tola A., Peru*

The Committee will assign to its members the duties of vice president, secretary, and such other offices that are deemed necessary. Likewise it is given authority to designate replacements in cases of retirement.

C. Committee membership will require annual minimum dues of \$100 per country, paid by a body or organization which (in the judgment of the Committee) would be representative of the activities which they promote in the respective countries.

D. The Committee will solicit the support of organizations and bodies, which by their character and goals, correspond to the objectives of the Committee and the activities which it advocates.”⁹

The recommendations were sent to the Ministries of Education, universities, and educational institutions in each country, as well as international organizations such as the OAS and UNESCO other institutions related to mathematics teaching and research.

The recommendations were divided into five parts which are summarized below:

I. Secondary school curriculum.

Here topics were proposed globally for the curriculum. They are outlined below:

For youths 12 to 15 years old: Sets, relations, whole numbers, binary operations, introduction of the axioms of geometry, introduction to rational

and real numbers, vector space of the plane, coordinates, ways of representing functions, metric geometry of the plane, scalar product, analytic geometry in orthogonal bases, systems of linear equations.

For youths 15 to 18 years old: study of real numbers, Euclidean space, orthogonal bases, Cauchy-Schwartz inequality, linear transformations of the plane, complex numbers, trigonometry, combinatoric analysis, the Euclidean algorithm, polynomials, some topological concepts, continuous functions, limits, sequences, derivatives, integration, special elementary functions, determinants, three-dimensional geometry, elementary probability and statistics.

Also, the following observations were added about the programs:

- a) It is advisable experimenting first in pilot courses.
- b) Topics should be put in an appropriate order.
- c) The program is for academic secondary education but it could be modified for technical and commercial schools.
- d) It is necessary for primary schools to prepare students for this program.

It was also proposed that studies be carried out in various countries to try to determine the results obtained in trials of the program, and that the programs be adapted for engineers and other applied fields.

Other resolutions.

II. The preparation of mathematics teachers for secondary schools and the first years of university.

It was most important to solicit the collaboration of the universities in this process. Also it was requested that formal agreements of collaboration be arranged among universities and that there be efforts to prepare more and better teachers.

III. In-service training of teachers.

Courses and other professional development activities should be intensified and, to the extent possible, permanent teacher training centers should be established.

IV. Preparation of textbooks and other bibliographic materials.

Great efforts should be made to try to publish monographs, student textbooks, public information pamphlets, pedagogical bulletins, a Latin American journal.

V. Various matters.

IACME was charged with aiding in the formation of local committees in each country; developing and disseminating a guide to Latin American institutions that offered high level programs in the field of mathematics; organizing periodic national meetings; organizing periodic national and regional colloquia to provide intensive courses on special topics; convening seminars on mathematical topics and on problems with teaching, and in which short communications on research projects could be presented and discussed. National societies of mathematics, in which secondary and university teachers take part, should be organized in order to promote the development of mathematics.

As can be deduced from the above summary, there was still interest in continuing to reform the teaching of mathematics in the American countries. There was also interest in continuing that reform along the same lines, that is, introducing "modern mathematics" into secondary education and teacher preparation programs. The recommendations are very clear in that sense. They are even more specific than the recommendations of the *Bogotá Conference* given that in this meeting there was even given, with a certain amount of detail, the topics that were to be taught in secondary schools.

This *Conference* exhibited a fundamental difference with respect to the *First*. In the *Bogotá Conference* general ideas about modern mathematics were expressed, with an explanation of why it was important to introduce it into secondary schools and stressing the importance of involving all countries in the change. In some way modern mathematics was defined, its main topics were mentioned, as well as an explanation of the new topics were related. Finally, there was an attempt to convince the participants of the advantages of carrying out the reform. On the other hand, in the *Lima Conference* it was assumed that countries were already involved, in one way or another, in the reform and therefore reports were solicited. The addresses were on topics of a more operational nature, and not so much about the great ideas of the reform, but instead how certain processes were being carried out in places that had been able to advance the most, especially in that particularly difficult topic: the pre-service and in-service development of teachers. These differences, however, are quite logical if we keep in mind that they represent parts of the same process.

MARSHALL STONE

This brief description would not be complete without emphasizing the figure of Marshall Stone, the driving force behind the creation of IACME.

Marshall Harvey Stone was born in New York on the 8th of April of 1903. He was 16 when he entered Harvard and graduated *summa cum laude* in 1922. Before being a professor at Harvard from 1933 to 1946, he was a professor at Columbia (1925-1927), Harvard (1929-1931), Yale (1931-1933) and Stanford in the summer of 1933. Although a Harvard graduate and professor, he is best known for having converted the Mathematics Department of the University of Chicago, as its Chair, into one of the main mathematical centers in the world. He achieved that by contracting famous mathematicians, such as Andre Weil, S. S. Chern, Antoni Zygmund, Saunders MacLane, and Adrian Albert.¹⁰ Paul

Halmos, Irving Seal, and Edwin Spanier¹¹ were also contracted during the same period. According to Saunders MacLane, the Mathematics Department that Stone created was at that time "without a doubt the leading Mathematics Department in the country"¹², and probably, we should add, in the world.

The scientific achievements of Stone were many. When he arrived in Chicago in 1946, upon recommendation to the President of the University of Chicago by John von Neumann, he had already completed important works in various areas of mathematics: the spectral theory of adjoint operators in Hilbert space, and on the algebraic properties of boolean algebras in the study of rings of continuous functions. He is known for the famous Stone-Weierstrass Theorem, as well as for the Stone-Cech compactification. His most influential book was *Linear Transformations in Hilbert Space and their Applications to Analysis*. He was elected a member of the *National Academy of Sciences* of the USA in 1938, at the age of only 35. He was President of the *American Mathematical Society* from 1943-1944.

Although an *International Mathematical Union* (IMU) had formally existed since the turn of the century, Stone renewed it, actually recreating it in the Rome Assembly in 1952. Stone was the first President of the new *Union* from 1952 to 1954. He was a member of the *International Commission on Mathematical Instruction* (ICMI) from 1959 to 1962 and of the *International Council of Scientific Unions* (ICSU).

Stone had a strong personality and exhibited an extraordinary charisma that permitted him to achieve his objectives in the University of Chicago, and , also exercise a powerful influence over the international mathematics community.

It should be mentioned, especially, that Stone had a great liking for Latin America. Many students from Latin America directly benefitted from him (among them, Prof. José Joaquín Trejos Fernández, who was President of Costa Rica from 1966 to 1970).¹³ The best indication of his appreciation of the region was, nevertheless, his decisive involvement in building and sustaining IACME during so many years (as its President from 1961 to 1972).

Finally, it should be mentioned that Prof. Stone was very much influenced by the ideas in the research and teaching of the Bourbaki group.¹⁴ He adopted many of the orientations of that group with respect to the axiomatic and abstract foundations of Mathematics and Mathematics Education. One indication of the close relationship between Stone and the Bourbaki group was the presence in Chicago of Andre Weil, who was for many years the dominant figure of the group and one of the most brilliant mathematics researchers at the time. Weil was in Chicago from 1947 to 1958.

The strong reputation of Dr. Stone in the world mathematics community explains the international support that IACME had in its beginnings.

In 1983 President Reagan presented Professor Stone with the most prestigious scientific award in his country: the *National Medal of Science* for his synthesis of analysis, algebra and topology.

Professor Stone died on the 8th of January of 1989 in Madras, India. His imprint on the world of mathematics was profound and lasting, but, we should also emphasize that he had a very special impact in Latin America.

IACME and the mathematics teachers of this region can never forget nor fail to recognize the invaluable support, so frank and unselfish, that Professor Stone gave us for the development of our discipline.

DATA FROM THE FIRST CONFERENCE

Given the importance of the *First Conference*, it is interesting to take a look at some of the details of the program, the organization, and the sponsorship of the event.

Invited Keynote Addresses

Name	Country
Enrique Cansado	Chile
Sven Bundgaard	Denmark
Howard Fehr	USA
Marshall Stone	USA
Gustave Choquet	France
Laurent Schwartz	France
Guillermo Torres	Mexico
Laurent Pauli	Switzerland

Program of Presentations¹⁵

Title of the Presentation	Presenter	Country
Mathematics and Our Technological Society	Alberto González	Argentina
Modern Applications of Mathematics	Enrique Cansado	Chile
Reform of the Teaching of Geometry	Howard Fehr	USA
The Formation of Mathematics Teachers	Luis Santaló	Argentina
The Preparation of Mathematics Teachers	Omar Catunda	Brazil
Mathematics Education in Latin America (Commentary introducing a Round Table)	Rafael Laguardia	Uruguay
The New Mathematics and its Teaching	Gustave Choquet	France

The beginnings: the first two conferences

Some Tendencies in Modern Mathematics	Marshall Stone	USA
Some Ideas about Teaching University Math	Guillermo Torres	Mexico
New Ideas in Teaching Math in US Colleges	E.J. McShane	USA
The Math Program in Swiss Secondary Schools	Laurent Pauli	Switzerland
The Mathematics Program in Denmark	Sven Bundgaard	Denmark
The Role of Math in Physics	Laurent Schwartz	France

Participants

Country	Delegates
Argentina	Alberto González Domínguez Luis Santaló
Bolivia	Moisés Artega
Brazil	Omar Catunda Alfredo Pereira Gómez
Canada	A. John Coleman
Colombia	Arturo Ramírez Montúfar
Costa Rica	Bernardo Alfaro
Chile	Enrique Cansado
Ecuador	José Rubén Orellana
El Salvador	Rodolfo Morales
Guatemala	Jorge Arias
Honduras	Edgardo Sevilla
Mexico	Marcelo Santaló
Nicaragua	Armando Hernández
Panama	Ramón Saavedra
Peru	José Tola Pasquel
Puerto Rico	Francisco Garrido
USA	E.J. McShane E.G. Begle
Uruguay	Rafael Laguardia
Venezuela	Manuel Balanazat
West Indies	L.R. Robinson

International Organizing Committee

Name	Country
Marshall Stone, President	USA
Howard Fehr, Secretary	USA
Marcelo Alonso	USA
José Babini	Argentina
Pablo Casas	Colombia
Leopoldo Nachbin	Brazil
Guillermo Torres	Mexico

Local Organizing Committee

Pablo Casas, President
Germán Zabala, Coordinating Secretary
Arturo Camargo
Otto de Greiff
Carlo Federici
Joaquin Giraldo Santa
Arturo Ramírez Montúfar
Alberto Schotborgh
Henri Yerly

Sponsoring Organizations

Organization of American States (OAS)
United Nations Educational, Scientific and Cultural Organization (UNESCO)
Ford Foundation of the USA
Rockefeller Foundation of the USA
National Science Foundation of the USA (NSF)
Association of Colombian Universities

Official Observers

Name	Organization Represented
Marcelo Alonso	OAS
Bowen C. Dees	NSF
Sanborn Brown	International Union of Pure and Applied Physics
Marshall Stone	Organization for Economic Cooperation and Development

Max Kramer	SMSG
Oscar Doderá Luscher	UNESCO
Mariano García Rodríguez	University of Puerto Rico

DATA FROM THE SECOND CONFERENCE

We provide here some data of interest on general aspects of the conference.

International Organizing Commission

Name	Country
César Abuaud	Chile
Bernardo Alfaro	Costa Rica
Howard Fehr, Executive Secretary of the Conference	USA
Carlos Imaz	Mexico
Rafael Laguardia	Uruguay
Leopoldo Nachbin	Brazil
Alfredo Pereira, Secretary of IACME	France
José Reategui, President of the Local Commission	Peru
Marshall Stone, President of IACME	USA
Alberto González	Argentina
José Tola, Vice President of IACME	Peru
Andrés Valeiras	Uruguay
Renato Völker	Argentina

Local Organizing Committee

Francisco Miró, *Honorary President*
José Reategui, *President*
José Luis Krumdieck, *Vice President*
César Carranza, *Secretary*
Victor Latorre, *Treasurer*
Jorge Sáenz, *Pro-Secretary*
Jorge Mendoza
José Ampuero
Antonio Baxeiras
Oscar Jahnsen
Alfredo Miró
Rubén Muñoz

Gerardo Ramos
Hugo Saravia

European Participants

Name	Country
Hans-Georg Steiner	Germany
Georges Papy	Belgium
Erik Kristensen	Denmark
Pedro Abellanas	Spain
Salvador Llopis	Spain
André Revuz	France

There were 41 participants from American countries.

Official Observers

Name	Country or Representation
Paul Dedecker	Belgium
Lidia Lamparelli	Brazil
Kleber Cruz	Brazil
Augusto Wanderley	Brazil
Ralph Fields	Columbia Teachers College Team
Sidney Grant	Columbia Teachers College Team
María Luisa Chavarría	Costa Rica
Alfonso Azpeitia	CSUCA
Francisco Jimenes	CSUCA
Enrique Cansado	Chile
Wade Ellis	NSF
Peter Faenkel	Ford Foundation
Heitor de Souza	OAS
Eugene Northrup	Ford Foundation - Turkey
Oscar Dodera	UNESCO

Supporting Organizations

Ford Foundation
National Science Foundation (NSF)
School Mathematics Study Group (MSG)
Institute of Pure and Applied Mathematics, National Engineering
University-Peru
Ministry of Public Education of Peru
OAS
UNESCO

Program of Presentations

Title of the Presentation	Presenter	Country
Some Observations on the Development of Mathematics in Latin America	Rafael Laguardia	Uruguay
Problems Encountered in the Reform of Mathematics with respect to Teachers and Programs	Luis Santaló	Argentina
Problems in Developing Mathematics Research in Latin America	José Tola	Peru
Studies for the Reform of Mathematics Teaching in Spain	Pedro Abellanos	Spain
Advances in Mathematics in Chile	César Abuaud	Chile
Progress in Mathematics Teaching in Brazil	Osvaldo Sangiorgi	Brazil
Activities of the OAS in Mathematics	Andrés Valeiras	Uruguay
An Experiment to Reconstruct the Mathematics Curriculum of the Secondary School	Howard Fehr	USA
Programs of Mathematics in the Teaching of Engineering	Carlos Imaz	Mexico
The Danish Mathematics Program	Erik Kristensen	Denmark
Efforts in Turkey to Improve Mathematics and Science Teaching in Secondary Schools Foundation	Eugene Northrup	Ford
The State of Reform of Mathematics Teaching in Belgium, 1966	Georges Papy	Belgium
Programs of Analysis	André Revuz	France
Programs of Analysis in Central American Universities	Eduardo Suger	Guatemala
The Retraining of Teachers in Puerto Rico	Mariano García	Puerto Rico
The Training of Teachers in Brazil	Martha de Souza	Brazil
A Rigorous Program for Preparing Teachers in West Germany	Hans-Georg Steiner	Germany

The New Programs and the Preparation of Teachers in Argentina	Renato Völker	Argentina
Preparation of Teachers of Mathematics for Secondary Schools	Luis Santaló Renato Völker	Argentina

NOTES

¹ *Educación Matemática en las Américas*, p. 45.

² *Ibid*, p. 86.

³ *Ibid*, p. 87-88

⁴ A Spaniard who resides in Argentina. One of the great mathematics teachers on the continent.

⁵ *Ibid.*, p. 65.

⁶ Stone, Marshall. "La Tarea de la Conferencia" [The Task of the Conference]. *Educación Matemática en las Américas II*, Lima, 1966, p. 8.

⁷ In 1962 the new secondary mathematics program was published; seminars were held in 1963 and 1964; summer institutes were held in 1964 and 1965; in 1965 a seminar was on held on teaching basic science; in 1964 a national in-service program for primary and secondary teachers was created.

⁸ Valeiras, Andrés. "Actividades de la Organización de los Estados Americanos en Matemática" [Activities of the Organization of American States in Mathematics]. *Educación Matemática en las Américas II*. Lima, 1966.

⁹ *Educación Matemática en las Américas II*, Lima, 1966, p. 3021-302.

¹⁰ See the article by Felix Browder, "The Stone Age of Mathematics on the Midway", in the book edited by Peter Duren: *A Century of Mathematics in America* (Vol. II), Providence, Rhode Island: AMS, 1989.

¹¹ Cf. MacLane, Saunders, "Mathematics at the University of Chicago: A Brief Story", in Duren, op. cit.

¹² MacLane locates the "Stone era" between 1946 and 1960, even though Stone resigned as Chair in 1952. MacLane, himself, followed Stone until 1958, and continued many of the activities begun by Stone.

¹³ The Department offered scholarships to students from 1948 to 1960. 114 PhDs graduated thanks to the scholarships. Among them is the famous Argentine mathematician, A.P. Calderón.

¹⁴ Cfr. Browder, op. cit.

¹⁵ In the order they were presented.