THE CURVATURE TRANSPARENCY AND THE ELECTROMAGNETIC FIELD TRANSFER OPERATORS IN THE CONCEPTUAL ELABORATION OF THE DIFFRACTION PHENOMENON

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Abstract

The free space electromagnetic field propagation in the fractional Fourier transform approach is used to show a form pedagogically elaborated for Fresnel and Fraunhofer diffraction concepts with the help of the curvature transparency and electromagnetic field transfer operators.

I. INTRODUCTION

One of the concepts of more importance in difractive optics maybe the referred to the propagation of electromagnetic waves in the free space; in particular in the mathematical formulation of the problem have been used two standard criteria: the corresponding to the paraxial approach which is perfectly developed by Goodman[1] and the second make reference to the metaxial approach, which was proposed and developed by Bonnet[2].
An important note is connected with the conceptual appropriation of the Fresnel and Fraunhofer diffraction phenomena by the students; because they have big difficulties in the particular treatment of the problems referred to the propagation in the free space.
In recent years a new tool for the description of the propagation appears in the context of the difractive optics and the propagation of electromagnetic field, which is known as the fractional Fourier transform. The present work uses this mathematical transformation with the objective of proposing a new pedagogic method through two basic operators, adjusting the theoretical construction way by the students and therefore improving the teaching and the correct conceptualisation of the diffraction phenomenon.

II. THE FRESNEL DIFRACTION BETWEEN PLANE EMITTERS AND PLANE DETECTORS.

When a planar object is illuminated with an plane wave, the field complex amplitude over the plane detector $U_F(\vec{r}\,')$ is obtain by Fresnel diffraction[1]:

$$U_F(\vec{r}\,') = \frac{-i}{\lambda D} \exp\left[\frac{-i\pi \vec{r}\,'^2}{\lambda}\left(-\frac{1}{D}\right)\right] \int \exp\left[\frac{-i\pi \vec{r}^{\,2}}{\lambda}\left(-\frac{1}{D}\right)\right] \exp\left[\frac{2i\pi \vec{r}.\vec{r}\,'}{\lambda(-D)}\right] U_A(\vec{r}) d\vec{r} \quad (1)$$

The time origin on *P* is shifted with respect to the time origin on *A*, then the factor $\exp\left[\frac{-2i\pi D}{\lambda}\right]$ has been neglected.

Using the definition of the fractional Fourier transform by Namias[3], (1) becomes:

$$U_F(\vec{r}\,') = \frac{-i}{\lambda D} \frac{\sqrt{2\pi \text{ sen}\,\alpha}}{\exp\left[i\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)\right]} \exp\left[\frac{i\pi \vec{r}\,'^2}{\lambda}\left(\frac{\text{sen}^2\,\alpha}{D}\right)\right] TFF^{\alpha}\left[U_A(\vec{r})\right] \quad (2)$$

Now, the field complex amplitude over the plane detector $U_F(\vec{r}\,')$ is proportional to the fractional Fourier transform of order α of the field complex amplitude of the planar object $U_A(\vec{r})$; this result is according to obtained by Pellat-Finet[4].

III. THE FRESNEL DIFRACTION BETWEEN SPHERICAL EMITTERS AND SPHERICAL DETECTORS.

In this case the illumination is with divergent spherical wave of radius R_A and observation on convex spherical detector of radius R_F.

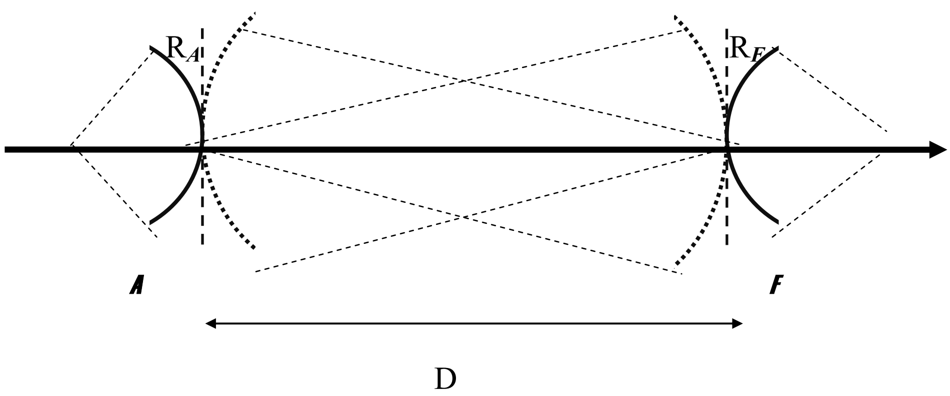

Fig. 1. Fresnel diffraction of the divergent spherical surface **A** observed on the convex spherical surface **F**.

In this case the amplitude distribution on the spherical detector in terms of the amplitude distribution of the spherical emitter is:

$$U_{F}\left(\vec{r}'\right)=\frac{-i}{\lambda D}\frac{\sqrt{2\pi\,\mathrm{sen}\alpha}}{\exp\left[i\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\right]}\exp\left[\frac{-i\pi\vec{r}'^{2}}{\lambda}\left(\frac{1}{R_F}-\frac{D+R_A\,\mathrm{sen}^2\alpha}{D\left(D+R_A\right)}\right)\right]TFF^{\alpha}\left[U_{A}\left(\vec{r}\right)\right] \quad (3)$$

If the illumination is with convergent spherical wave of radius R_A and observation is made over concave spherical detector of radius R_F, R_A and R_F in (3) should be changed for $-R_A$ and $-R_F$ respectively.

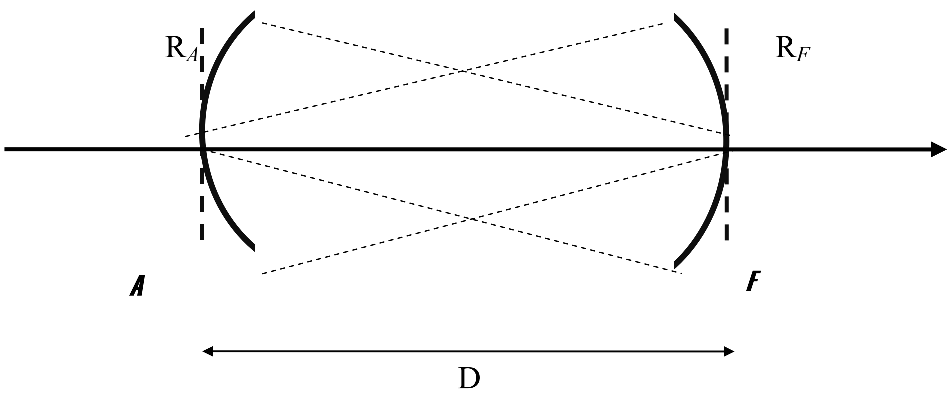

Fig. 2. Fresnel diffraction of the convergent spherical surface **A** observed on the concave spherical surface **F**.

These results are according to obtained by Ozaktas and Mendlovic[5].

IV. TRANSFORMATION OPERATORS[2]

4.1. Curvature Transparency Operator.

If $D \to 0$ and $\alpha = 0$ then (3) writes:

$$U_{F}\left(\vec{r}\right)=U_{A}\left(\vec{r}\right)\exp\left[\frac{-i\pi\vec{r}^{\,2}}{\lambda}\left(\frac{1}{R_F}-\frac{1}{R_A}\right)\right] \quad (4)$$

Which corresponds to a transparency since modulus of transfer is equal to 1.

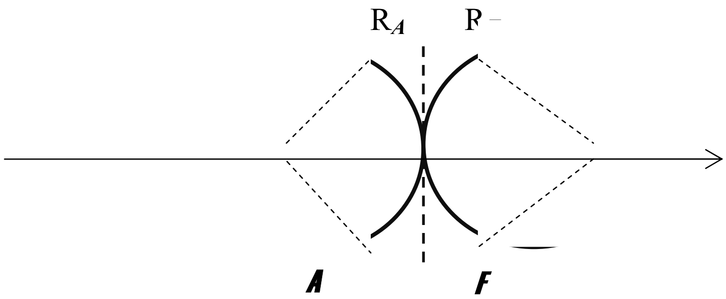

Fig. 3. Fresnel diffraction of the spherical surface **A** observed on the spherical surface **F** when D tends to zero.

Therefore the step of **A** to **F** can express by means of:

$$\boldsymbol{t}_{\mathrm{FA}}\left(\vec{r}\right) = \exp\left[\frac{-i\pi\vec{r}^{\,2}}{\lambda}\left(\frac{1}{R_F} - \frac{1}{R_A}\right)\right] \tag{5}$$

This interesting result can be interpreted this way:
"The field transfer of a spherical emitter **A** *to a tangent sphere* **F** *is expressed by a factor of quadratic phase depending on the respective curvature radio of* **A** *and* **F***, of agreement with the equation (5)".*

4.2. Electromagnetic Field Transfer Operator.

If $\alpha = \frac{\pi}{2}$, Then (3) writes:

$$U_{\boldsymbol{F}}\left(\vec{r}^{\,'}\right) = \frac{-i\sqrt{2\pi}}{\lambda D}\exp\left[\frac{-i\pi\vec{r}^{\,'2}}{\lambda}\left(-\frac{1}{R_F} - \frac{1}{D}\right)\right]TF\left[U_{\boldsymbol{A}}\left(\vec{r}\right)\right]$$

If $R_F = -D$, we obtain the Fourier sphere[2]:

$$U_{\boldsymbol{F}}\left(\vec{r}^{\,'}\right) = \frac{-i\sqrt{2\pi}}{\lambda D}TF\left[U_{\boldsymbol{A}}\left(\vec{r}\right)\right] \tag{6}$$

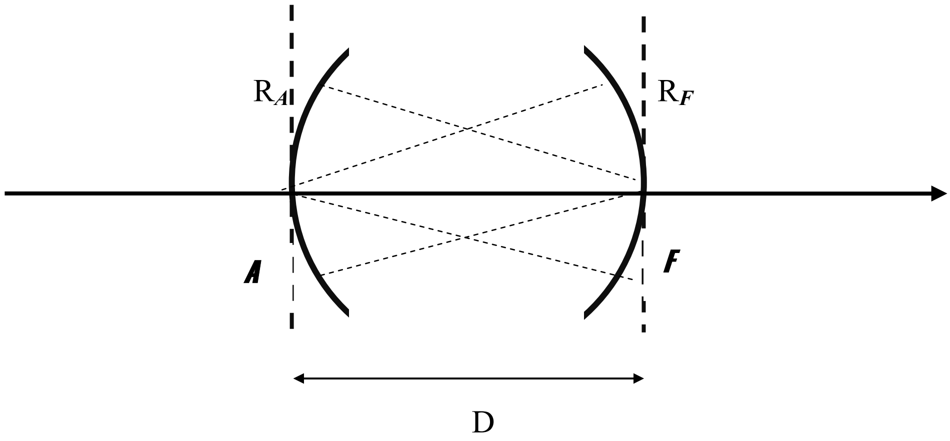

Fig. 4. Fraunhofer diffraction of the spherical emitter **A** observed on spherical surface **F**. **F** is called the Fourier sphere of **A**.

As the field amplitude in **F** is the Fourier standard transform of the field amplitude in **A**; the sphere **F** is denominated "sphere of Fourier of **A**", corresponding physically to the diffraction of Fraunhofer. This important conclusion can be expressed in this way:

"Be **A***, a spherical emitter with radio $R_A = D$ and* **F** *the sphere whose radio is $R_F = -D$, centred on* **A***. The field transfer of* **A** *to* **F** *corresponds to the Fraunhofer diffraction, it is expressed mathematically by means of the equation (6)."*

It is important to write down that these results are completed for propagation of waves in the free space, supplementing the results obtained previously where lens are used to reach the fractional Fourier transform[6].

V. THE COMPOSITION OF OPERATORS.

To show the way to act of the two operators described in this work, we proceed to apply them to the configuration of the figure 5.

In this case it is looked for obtain the amplitude distribution of the electromagnetic field on the spherical detector of radio R_F produced by the electromagnetic field amplitude of the spherical emitter of radio R_A.

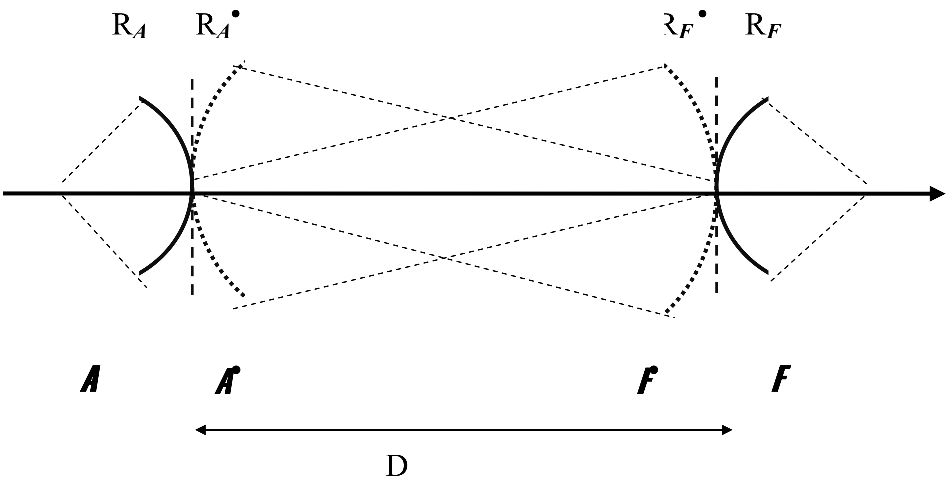

Fig. 5. Determination of the field transfer **A** from to **F**.

Step I

Curvature transparency operator is applied among the spherical surfaces **A** and **A•**; then (4) writes:

$$U_{A^\bullet}(\vec{r})=U_{A}(\vec{r})\exp\left[\frac{-i\pi\vec{r}^{\,2}}{\lambda}\left(\frac{1}{R_A^\bullet}-\frac{1}{R_A}\right)\right] \tag{7}$$

With $R_A^\bullet = D$

Step II

Electromagnetic field transfer operator is applied from **A•** to **F•**; then (6) becomes:

$$U_{F^\bullet}(\vec{r}\,')=\frac{-i\sqrt{2\pi}}{\lambda D}TF\left[U_{A}(\vec{r})\exp\left[\frac{-i\pi\vec{r}^{\,2}}{\lambda}\left(\frac{1}{D}-\frac{1}{R_A}\right)\right]\right] \tag{8}$$

Step III
Transfer from **F•** to **F**. Curvature transparency operator is applied among **F•** and **F**; then (4) writes:

$$U_{F}(\vec{r}\,')=U_{F^\bullet}(\vec{r}\,')\exp\left[\frac{-i\pi\vec{r}\,'^{2}}{\lambda}\left(\frac{1}{R_F}-\frac{1}{R_F^\bullet}\right)\right] \tag{9}$$

With $R_F^\bullet = -D$

Finally, the total transfer can be easily written as:

$$U_F\left(\vec{r}\,'\right)=\frac{-i\sqrt{2\pi}}{\lambda D}\exp\left[\frac{-i\pi}{\lambda}\left(\frac{1}{R_F}+\frac{1}{D}\right)\vec{r}\,'^2\right]\int_A \exp\left[\frac{-i\pi}{\lambda}\left(\frac{1}{D}-\frac{1}{R_A}\right)\vec{r}^{\,2}\right]\exp\left[\frac{2i\pi}{\lambda D}\vec{r}.\vec{r}\,'\right]U_A(\vec{r})d\vec{r} \qquad (10)$$

VI. CONCLUSIONS

It has been shown as the propagation of electromagnetic waves you can express through single two operators denominated curvature transparency and electromagnetic field transfer (Fourier sphere). This result facilitates the proposition of an alternative method for the solution of problems in difractive optics, as well as allows obtaining a better handling on behalf of the students of the physical interpretation of the electromagnetic field propagation and the Fresnel and Fraunhofer diffraction.

IV. ACKNOWLEDGEMENT

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